B. Tech.
(SEM. III) (ODD SEM.) EXAMINATION, 2009-10
MATHEMATICS - III

Time : 3 Hours] [Total Marks : 100

1. Attempt any four of the following : 5x4

(a) Find Fourier integral of \( f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \)

(b) Use Fourier sine Transform to solve the equation
\[ \frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2} \]
under the conditions
(i) \( u(0, t) = 0 \)
(ii) \( u(x, 0) = e^{-x} \)
(iii) \( u(x, t) \) is bounded.

(c) Find Z-transform of \( C^k \cos(k \alpha) ; \ K \geq 0 \).

(d) Solve the difference equation
\[ y_{k+2} - 2 \cos \alpha y_{k+1} + y_k = 0 ; \ y_0 = 0, y_1 = 1, \]
using Z-transform.

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[Contd...]
(e) Find the Fourier sinc transform of

\[ f(x) = \frac{e^{-ax}}{x}, \quad a > 0 \]

(f) Find the Fourier transform of

\[ f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \]

2. Attempt any four of the following:

(a) If \( f(z) = \frac{x^3y(y-ix)}{x^6+y^2}, \quad z \neq 0 \) and \( f(0) = 0 \), show that \( \left( \frac{f(z) - f(0)}{z - 0} \right) \to 0 \) as \( z \to 0 \) along any radius vector but not as \( z \to 0 \) in any manner.

(b) Prove that the function \( \sinh z \) is analytic and find its derivative.

(c) Evaluate \( \int_{C} \left( \frac{z^2 + 1}{z^2 - 1} \right) dz \); \( C \) is (i) \( |z| = \frac{3}{2} \)

(ii) \( |z - 1| = 1 \) (iii) \( |z| = \frac{1}{2} \)

(d) If \( u = (x-1)^3 - 3xy^2 + 3y^2 \), determine \( \nu \) so that \( u + i\nu \) is a regular function of \( x + iy \).
(e) If φ and ψ are functions of x and y satisfied
Laplace equation, prove that \( S + it \) is
holomorphic where \( S = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \) and
\[ t = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} \].

(f) Prove that the function \( f(z) = z|z| \) is not
analytic anywhere.

3. Attempt any two parts: 10x2

(a) Determine the poles of the function
\[ f(z) = \frac{z^2}{(z-1)^2(z+2)} \] and residue at each
poles. Hence evaluate \( \int_C f(z)\,dz \); \( C:|z|=3 \).

(b) Find the Laurent's expansion for
\[ f(z) = \frac{7z-2}{z(z+1)(z-2)} \] in the region given by
(i) \( 0 < |z+1| < 1 \) (ii) \( 1 < |z+1| < 3 \)
(iii) \( |z+1| > 3 \).

(c) Evaluate \( \int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} \); \( a > b > 0 \)

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4 Attempt any two parts: 10x2
(a) Fit a Poisson distribution to the following data and calculate theoretical frequencies:

<table>
<thead>
<tr>
<th>Deaths</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>122</td>
<td>60</td>
<td>15</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

(b) Find the correlation coefficient between \( x \) and \( y \) for the following data:

<table>
<thead>
<tr>
<th>( x )</th>
<th>60</th>
<th>34</th>
<th>40</th>
<th>50</th>
<th>45</th>
<th>41</th>
<th>22</th>
<th>43</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>75</td>
<td>32</td>
<td>34</td>
<td>40</td>
<td>45</td>
<td>33</td>
<td>12</td>
<td>30</td>
</tr>
</tbody>
</table>

(c) The equations of two regression lines obtained in a correlation analysis of 60 observations are
\[ 8x - 10y + 66 = 0, \quad 40x - 18y = 214 \]
and variance of \( x = 9 \). Calculate:
(i) \( \bar{x} \) and \( \bar{y} \)
(ii) standard deviation of \( y \)
(iii) coefficient of correlation.

5 Attempt any two parts:
(a) Fit the second degree parabola to the following data:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1090</td>
<td>1220</td>
<td>1390</td>
<td>1625</td>
<td>1915</td>
</tr>
</tbody>
</table>

(b) Solve \( 28x^3 - 9x^2 + 1 = 0 \) using Cardon's method.

(c) Solve \( x^4 - 10x^3 + 44x^2 - 104x + 96 = 0 \) using Ferrari's method.