B. TECH. IIInd - Year
THIRD SEMESTER EXAMINATION, 2007-2008
TMA-301 MATHEMATICS - III

Time : 3 Hours
Maximum Marks : 100

Note: (i) Attempt any FIVE questions.
(ii) All questions carry equal marks.

1. (a) Express the function \( f(x) = \begin{cases} 1, & \text{for } |x| \leq 1 \\ 0, & \text{for } |x| > 1 \end{cases} \) as a Fourier Integral. Hence evaluate the value of

\[
\int_{0}^{\infty} \frac{\sin \lambda \cos \lambda}{\lambda} x \, d\lambda \tag{7}
\]

(b) Find the Fourier Sine Transform of \( \frac{e^{-ax}}{x} \), \( a > 0 \). \( \tag{7} \)

(c) Solve by Z-Transform \( y_{k+2} - 4y_k = 0 \), given that \( y_0 = 0, y_1 = 2 \). \( \tag{6} \)

2. (a) Determine an analytic function \( f(z) \), in terms of \( z \) whose real part is \( e^{-x} (x \sin y - y \cos y) \). \( \tag{7} \)

(b) If \( f(z) \) is a regular function, show that

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2 \tag{7}
\]
(c) Find the bilinear transformation which maps the points \( z = 0, -1, i \) into \( w = i, 0, 1 \).

3. \( (a) \) Evaluate the following integrals:

\[
(i) \quad \int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z + 1)(z + 2)} \, dz,
\]
where \( C \) is the circle \( z = 3 \).

\[
(ii) \quad \int_C \frac{z^2 + z + 1}{(z^2 - 1)(z - 2)} \, dz,
\]
where \( C \) is the circle \( z = 3/2 \).

\( (b) \) Using the residue theorem, evaluate

\[
\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 1)(x^2 + 4)} \, dx
\]

4. \( (a) \) Expand \( f(z) = \frac{1}{(z - 1)(z - 2)} \) in Laurent series valid for

\( (i) \ 0 < z < 1, \quad (ii) \ 1 < z < 2, \quad (iii) \ z > 2. \)

\( (b) \) Find the image of \( z - 3i = 3 \), under the mapping \( w = (1/z) \).

\( (c) \) Find the poles of the following function. Find the order of each pole and residue at it:

\[
f(z) = \frac{e^{2z}}{(1 + e^z)}
\]
5. (a) Find the relation between moment about the mean and moment about any arbitrary point. The first four moments of a distribution about the value 4 of the variate are $-1.5$, $17$, $-30$ and $108$. Calculate the first four moments about the mean and find $\beta_1$ and $\beta_2$.

(b) In a partially destroyed laboratory record of an analysis of correlation data, the following results are only eligible:

Variance of $x = 9$, Regression lines are $8x - 10y + 66 = 0$ and $40x - 18y = 214$. What were:

(i) The mean value of $x$ and $y$,
(ii) Standard deviation of $y$,
(iii) The correlation coefficient of $x$ and $y$.

6. (a) Solve the equation, 
\[ x^4 - 10x^3 + 35x^2 - 50x + 24 = 0 \]
by Ferrari method.

(b) Fit a second degree parabola to the following data:

<table>
<thead>
<tr>
<th>X</th>
<th>0.0</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1.0</td>
<td>4.0</td>
<td>10.0</td>
<td>17.0</td>
<td>30.0</td>
</tr>
</tbody>
</table>

7. (a) In a normal distribution, $31\%$ of the items are under 45 and $8\%$ are above 64. Find the mean and standard deviation.

(b) In a poisson distribution, if
\[ P(X = 2) = 9P(X = 4) + 90P(X = 6), \]
then find $P(X = 1)$

(c) Find the recurrence formula for the binomial distribution.